

P425/1
PURE
MATHEMATICS
Paper 1
July / Aug. 2022
3 hours



UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES.

Answer all questions in section A and any five from section B.

All necessary working must be shown clearly.

Silent non-programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section B will not be marked.

SECTION A (40 MARKS)

Answer ALL questions in this section

- Solve the equation: $1 - \cos 2\theta = \sin \theta$ for $90^\circ \leq \theta \leq 180^\circ$ (05 marks)
- Given that $\frac{92}{(z+1)^2} = a + b_1$, find the real numbers a and b . (05 marks)
- Show that the stationary points of the curve $y^2 = x^2 + 2xy + 11$ lie on the line $y = -x$. State the x -coordinates of the points. (05 marks)
- The acute angle between the lines $2y - x - 3 = 0$ and $y = px + 3$ is 45° . Find the possible values of p . (05 marks)
- Calculate the distance of the origin $O(0, 0, 0)$ from each of the planes $3x - 4y + 12z + 13 = 0$ and $3x - 4y + 12z - 37 = 0$; hence deduce the distance between the planes. (05 marks)
- Evaluate $\int_0^{1/2} \frac{92}{1-x^2} dx$ (05 marks)
- The first, second and fourth terms of an arithmetic progression form a geometrical progression. Find the common ratio of the G.P. (05 marks)
- The volume V of a cone varies such that the height, h , of the cylinder is twice its base radius.
 - Show that $V = \frac{\pi}{12} h^3$
 - Find the rate at which V changes with height, at the instant when $h = 4\text{cm}$. (05 marks)

SECTION B (60 MARKS)

9.

A curve is represented by the parametric equations:

$$x = t^2 - 2t - 3$$

$$y = t^2 + 2t - 3 \quad \text{where } t \text{ is the parameter.}$$

Find: (a)

$$\frac{dy}{dx} \text{ and } \frac{d^2y}{dx^2}$$

in terms of t , hence find and determine the

nature of the stationary points of the curve. (08 marks)

(b)

the equation of the tangent to the curve at the point where the curve cuts the positive y -axis. (04 marks)

10.

A circle whose centre is $C(1, 6)$ touches the line $y = \frac{3}{4}x - 1$ at point A. (05 marks)

Find the:

(a) equation of the circle. (06 marks)

(b) coordinates of point A. (06 marks)

11.

(a) Solve the equation: $3\sin\theta - 4\cos\theta = 4$ for $0^\circ \leq \theta \leq 360^\circ$ (06 marks)

(b) Prove that: $\frac{\sin 3A - \sin A}{2\cos A + \sec A} = \frac{1}{4} \sec^2 A$ (06 marks)

12.

(a) Expand $\sqrt{4 - 3x}$ up to the term in x^3 , hence find the error made in using $x = 1$ in the expansion. (07 marks)

(b) Evaluate $\sqrt{61}$ to 4 decimal places. (03 marks)

13.

The tangent to the curve $y = x^3$ at the point $A(-1, -1)$ meets the curve again at point B. (07 marks)

(a) Find the:

(i) equation of the tangent at A. (07 marks)

(ii) coordinates of point B. (05 marks)

(b) Calculate the area bounded by the line AB and the curve. (05 marks)

14. The line $r = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ meets the plane $3x + 2y + z = 29$ in point A. Find the:
- coordinates of point A (07 marks)
 - acute angle between the line and the plane (05 marks)
15. (a) Given $Z_1 = -1 - i\sqrt{3}$, $Z_2 = -1 + i$, $Z_3 = 4i$; find the principal argument of $\frac{Z_1^3 Z_2^2}{Z_3}$. (06 marks)
- (b) Find, in Cartesian form, the cube roots of -8 . (06 marks)
16. (a) Solve: $\sin x \frac{dy}{dx} + y \cos x = \tan 3x$ (05 marks)
- (b) The price P of a litre of petrol increases at a rate which is directly proportional to the price. If the price doubles every 10 days; find the percentage increase in the price after 20 days. (07 marks)

END